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A numerical–analytical method for solving the non-linear problem of the dynamics of heat transfer in channels

B. P. KOROL'KOV and E. A. TAIROV
Siberian Energy Institute, Siberian Branch of the U.S.S.R. Academy of Sciences, Irkutsk, 664033, U.S.S.R.

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Abstract—The ways of solving the non-linear problems of the dynamics of heat exchange processes are discussed. A numerical–analytical approach is suggested based on the analysis of the capabilities of physical and numerical experiments and analytical methods. Two numerical–analytical methods (the method of the 'frozen impulse-transient function' (FITF) and the 'step transient function' (STF) method) are worked out and described, which allow one, using the adaptation concept, to construct an approximate solution for the non-linear problem in terms of the functional relations obtained for the linear approximation. The STF method is justified experimentally and the error of theoretical results does not exceed 5%. The FITF method also gives good results for monotonous processes.

INTRODUCTION

THE PROPAGATION of thermal and hydrodynamic disturbances in non-adiabatic channels is of current interest for solving the applied problems of control, reliability and safety in power and chemical engineering, cryogenic technology and the food industry. In their full formulation, these problems are described by a non-linear system of differential mass conservation equations, the dimensionality of which is determined by the entire set of heat exchange media.

The spatial inhomogeneity of the developing processes essentially complicates the solution of these equations, and therefore most authors have used spatially one-dimensional models when treating extended channels. Here, along with numerical methods for analyzing non-stationary phenomena, the construction of analytical solutions becomes feasible in some cases. Among the indisputable advantages of the latter is the possibility of establishing explicit functional relations between effecting factors and sought variables, which accounts for the undiminished interest in analytical methods [1, 2].

However, the validity range of analytical approaches is much more narrow than that of numerical methods, especially if non-linear effects in occurring phenomena are taken into consideration.

The present study works out a numerical–analytical approach which, in the present authors' opinion, seems to be promising for solving non-linear transfer problems. This least developed method is expected to produce many novel interesting results, and their generalization is believed to eliminate the constraints of purely numerical and analytical methods.

In the previous works of the present authors [3, 4] the methods and tools for solving various linear problems of the dynamics of heat transfer were developed. The fundamental solutions (in the form of Green functions) obtained in these studies led to the unification of the search for the heat transfer system response to arbitrary effects, while the use of the weak coupling concept allowed the complex boundary-value problem of conjugate heat transfer to be reduced to the simpler problem of solving Volterra's second kind integral equations. To find quantitative results, effective computational algorithms are constructed. It is this positive (in the present authors' opinion) experience which was used as the basis of the numerical–analytical approach to solving non-linear heat transfer problems.

Many means of constructing solutions for the non-linear problem are available, among which the asymptotic and adaptive methods seem to be the most attractive.

The general basis for asymptotic methods is the construction of solutions in the form of a series, where the first term reflects the linear properties of the system and each subsequent term brings the non-linear solution closer to the exact one, being constructed on the basis of the already known preceding terms. Of the asymptotic methods applied for solving heat transfer problems, the perturbation method (the method of a low parameter) and the method of functional series (the Wiener and Volterra methods) have gained much recognition.

At present, the most promising method for solving the non-linear problem of the dynamics of heat transfer in a channel seems to be the adaptive method, which is realized in the present study.

By confining the discussion to heat transfer processes in normal (not emergency) situations it is possible to talk only about variations of the coefficients in

NOMENCLATURE

C — T_m/T_B
C_B, C_M — specific heat of flow and wall, respectively [kJ kg^{-1} K^{-1}]
D — flow rate [kg s^{-1}]
d — diameter [m]
E_{ij} — impulse transient function
f — cross-sectional area [m^2]
g — specific mass of substance [kg m^{-1}]
h — specific heat transfer surface [m]
h_{ij} — step transient function
i — enthalpy [kJ kg^{-1}]
K_B — $-K_q(\mathrm{d}i_0/\mathrm{d}z)$
K_p — $(-1/C_B)(\mathrm{d}i/\partial p)_t$
K_q — $1/h$
n — exponent on D in relation for 0
p — pressure [N m^{-2}]
q — linear heat flux density [kW m^{-1}]
S_0 — $T_M T_B/(T_M+T_B)$
t — flow temperature [K]
T_B — $g_B C_B K_q$
T_M — $g_M C_M K_q$
V — special function
z — spatial coordinate [m].

Greek symbols
α — heat transfer coefficient [kW m^{-2} K^{-1}]
Δ — dynamic deviation, $\Delta x = x - x_0$
ζ — hydraulic resistance coefficient of valve [m^{-4}]
η — dimensionless time
θ — wall temperature [K]
ξ — dimensionless spatial coordinate
ρ — density [kg m^{-3}]
τ — time [s].

Subscripts
B — flow
eff — effective
ex — exit
exp — experimental
fr — friction
in — inlet
M — wall
s — source
theor — theoretical
tr del — transport delay
0 — initial value.

conservation equations or of the parameters of integral operators associated with them; the structure of equations and their solutions are assumed to be invariant. The drift of coefficients is connected with their dependence on the regime parameters (fluid velocity, heat load, pressure). If the state of the system varies slowly, the idea naturally suggests itself of attempting to employ a linear model with continuously or directly rearranging coefficients for describing the unsteady heat transfer process.

For a certain time interval (the smaller this is, the more strident are the requirements for the solution accuracy) the coefficients can be assumed to be constant. In this case the model of the process in the vicinity of the fixed time instant can be regarded as linear with ample justification. All the virtual variations in the coefficients of the equations on the time interval under study are accumulated and at the instant of transition to the subsequent time interval are converted stepwise to new values, and so on. This method of solution is not rigorously substantiated and therefore the stand experiment is a very important factor.

MATHEMATICAL MODEL

Consideration will be given to an extended channel with the heat absorbing walls heated by an independent external heat flux. The assignment of the external heat flux does not exclude the presence of the third heat exchanging medium (fluid flux), but in some cases the possibility of substituting the equation of the supplementary circuit by the condition $q = q(z, \tau)$ can be proved.

The mathematical model describing the process of unsteady heat transfer and hydrodynamics is based on the fundamental mass, energy and momentum conservation laws written for interacting media. For formulating dynamics equations the generally accepted assumptions appropriate in such cases [5], will be introduced.

(1) Use is made of the parametric integral relations averaged over the channel cross-section, which corresponds well to the liquid turbulent flow characteristic of real regimes.

(2) The time constant for unsteady hydrodynamic and thermal boundary layers is much smaller than the time of variation of the process parameters and can, therefore, be neglected.

(3) The heat transfer agent velocity in the channel is much smaller than the local speed of sound, thus obviating the necessity to analyse acoustic effects.

(4) The variations of potential and kinetic energies in the energy equation for a heat transfer agent are disregarded owing to their smallness as against enthalpy change.

(5) The heat flux directed along the working medium motion is negligible in comparison with the radial heat flux.

(6) The physical properties of the wall material are taken at the mean wall thickness temperature.

(7) The pressure drop along the section is negligible as compared with its absolute value. For this reason, its effect on density and enthalpy is ignored.

Presenting the dependent variables as the sum of the stationary value and the dynamic deviation, and taking into account the statics equations, yields

$$\frac{\partial \Delta D}{\partial z}+f\frac{\partial \Delta \rho}{\partial \tau}=0$$

$$\Delta D\frac{\mathrm{d}i_0}{\mathrm{d}z}+D\frac{\partial \Delta i}{\mathrm{d}z}+f\rho\frac{\partial \Delta i}{\partial \tau}=\Delta\alpha h(\theta_0-t_0)+\alpha h(\Delta\theta-\Delta t)$$

$$\Delta q-g_{\mathrm{M}}c_{\mathrm{M}}\frac{\partial \Delta\theta}{\partial \tau}=\Delta\alpha h(\theta_0-t_0)+\alpha h(\Delta\theta-\Delta t)$$

$$\frac{1}{f}\frac{\partial \Delta D}{\partial \tau}+\frac{\partial}{\partial z}\left[\Delta\left(\frac{D^2}{\rho f^2}\right)+\Delta p+\Delta p_{\mathrm{fr}}\right]=0. \quad (1)$$

Here, the equation of heat propagation in the solid envelope of the channel is written in balance form and the motion equation in the form of hydraulic resistance.

The solution of this system gives the space–time variations, ΔD, Δi, $\Delta\theta$ and Δp, in the presence of external thermal and hydrodynamic disturbances, which are shown in the structural scheme in Fig. 1.

It is seen that the non-linearities in equation (1) are formed by the variable flow rate $D(z, \tau)$ and by the varying thermal physical properties of the heat transfer agent. This also applies to the system of closing relations which the basic equations should be augmented with:

$$\rho=\rho(t,p),\quad i=i(t,p)$$

$$\alpha=K_\alpha f_1(t,p)f_2(D)$$

$$p_{\mathrm{s}}-p_{\mathrm{in}}=\zeta\frac{D_{\mathrm{in}}^2}{\rho_{\mathrm{s}}}. \quad (2)$$

A complete model of the dynamics of heat transfer in the channel can be realized only with the aid of a computer. Two methods are possible here. One of them involves reducing differential equations to a system of algebraic equations of high dimensions with the use of finite difference methods. The other suggests that preliminary transformations be made to the original model, which allow the introduction of more

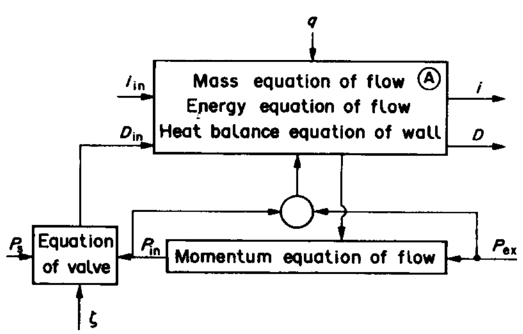

FIG. 1. Structural scheme of the calculation of a channel.

economical and obvious algorithms at the expense of a slight decrease in accuracy. This involves analytical transformations resulting in integral models. Here, the problem of increasing the computational time step is solved readily and naturally, thus enabling its selection to be guided only by the required completeness of the reproduction of the unsteady process curve. The cause and effect relationships of the phenomenon considered are revealed distinctly by the integral models via natural characteristic functions, i.e. impulse and step transient functions.

Next, insight will be offered into the problems of finding characteristic functions and of their application to the approximate calculations of transient processes in the case of strong arbitrarily varying disturbances.

LINEAR THEORY

First it will be noted that for the majority of the heat exchanger parts, the total pressure in a channel considerably exceeds its drop along the length, and this allows the integration of the first three equations (1) separately from the last one with allowance for the mean temperature variations in the channel. Here, the motion equation is necessary for predicting the pressure drop over the section under consideration and the pressure in the boundary section is opposite to that for which the pressure is specified. The structural scheme of the problem is given in Fig. 1. The crucial point here is the finding of the operators of direct signal transformation in the element A of the scheme. For this element, the variations in the flow rate $D_{\mathrm{in}}(\tau)$ and pressure $p(\tau)$ in the channel are assumed to be assigned functions, while closure by natural feedback is achieved in the solution algorithm of the boundary-value problem.

In solution it is convenient to employ the characteristic transient functions describing the laws of propagation of disturbances in the direction of the current.

Analytical integration of the equations yields different results depending on whether there is a single-phase flow or a flow of a boiling heat transfer agent. For the sake of definiteness, the flow will be considered single-phase and slightly compressible. The latter provision implies that the flow rate in any channel cross-section differs little from its value on the original coordinate, and this difference in the energy equation can be neglected, having taken $D(z, \tau) = D_{\mathrm{in}}(\tau)$ in it. This means that concentrated coefficients are used in the distributed model. The substitution of density, specific heat and heat transfer coefficient varying along the length by their mean integral values is substantiated in refs. [6–8]. Thus, the enthalpy variations $\Delta i(z, \tau)$ are determined from two energy equations at constant coefficients and linearized closing relations. It should be noted that to coordinate the dynamic deviations with the static increments a perturbed flow rate value should be taken as a coefficient in front of the derivative $\partial\Delta i/\partial z$.

The solution obtained for $\Delta i(z, \tau)$ is employed in integrating the continuity and motion equations.

The following expressions of impulse transient functions for the flow enthalpy are found from the analytical integration of the equations:

$$E_{ii} = \frac{\xi}{T_M \eta} V_{2,0} + \delta(\tau - \tau_{\mathrm{ir\,del}})\,e^{-\xi}$$

$$E_{iD} = \frac{K_B C_B}{T_M + T_B}[1 - V_1 - (1 - S_0 v)(e^{-S_0 \tau} - V_{1,C})]$$

$$E_{iq} = \frac{K_q C_B}{T_M + T_B}(1 - e^{-S_0 \tau} - V_1 + V_{1,C})$$

$$E_{ip} = \frac{K_p C_B}{T_B}(V_{1,C} - e^{-S_0 \tau}). \quad (3)$$

Here, the first index of the function E_{ix} denotes the exit parameter, the second index designates the inlet effect and special V functions are expressed in terms of the integrals of the Bessel functions [9].

Using the convolution theorem the following can be written:

$$\Delta i(z, \tau) = \sum_{K=1}^{4} \int_0^{\tau} E_{ix_K}(z, \tau - t, \boldsymbol{\sigma}_s) \Delta x_K(t)\, dt \quad (4)$$

where $\Delta x_K(\tau)$ are arbitrary perturbations $\Delta i_{\mathrm{in}}(\tau)$, $\Delta D(\tau)$, $\Delta q(\tau)$ and $\Delta p(\tau)$, and $\boldsymbol{\sigma}_s$ is the stationary vector of the dynamic system parameters. Having obtained, with allowance for solutions (3), the impulse transient functions of the form E_{Dx} from the first equation of system (1), a similar expression for the flow rate variations can be written

$$\Delta D(z, \tau) = \sum_{K=1}^{4} \int_0^{\tau} E_{Dx_K}(z, \tau - t, \boldsymbol{\sigma}_s) \Delta x_K(t)\, dt. \quad (5)$$

The analytical specification of the nuclei E_{ix} and E_{Dx} substantially simplifies the problem of calculating integrals (4) and (5).

The equation of motion within the computational spatial section is reduced here to the linearized algebraic form

$$\Delta p_{\mathrm{ex}} = f(\Delta p_{\mathrm{in}}, \Delta i, \Delta D). \quad (6)$$

Relations (4)–(6), augmented with the corresponding boundary-value conditions, fully represent the computational structural scheme given in Fig. 1. They naturally track the possible variations and augmentations in a particular physical system, since the inlet–exit relations can be used to describe each element of the scheme. A disadvantage of this approach to the dynamic system is the necessity of employing the iteration for solving the boundary-value problem.

This disadvantage is eliminated by transforming relations (4)–(6) into a system of two Volterra second kind integral equations, which were solved by the non-iterative method in ref. [4]. Also considered there was the application of the method of integral equations to solving the problem of dynamics in a steam generating channel represented in the form of three sequential sections differing in a phase state of the heat transfer agent. Figure 2 presents the computational and experimental curves of transient processes which correspond to the step perturbations of enthalpy, flow rate and heat input.

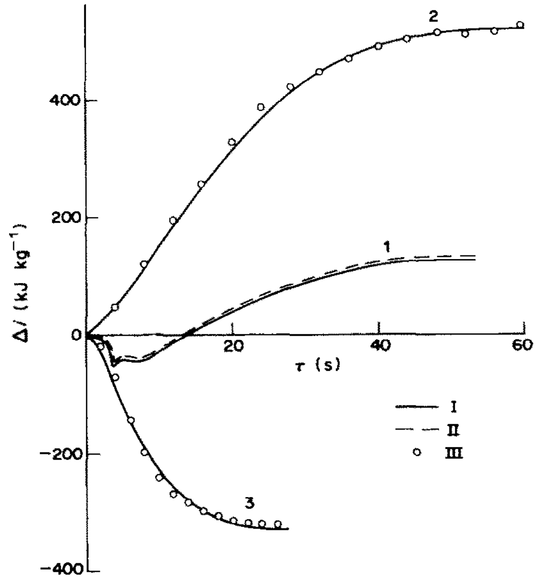

FIG. 2. Variations in the steam enthalpy during disturbances: of the inlet water temperature (1) ($\Delta i = 128$ kJ kg^{-1}); of the flow rate (2) ($\Delta D = -20\%$); of the external heat input (3) ($\Delta q = -12\%$). I, Calculation; II and III, experiment.

NON-LINEAR THEORY

The method of 'frozen' impulse transient function (FITF)

In the case of non-linear systems, the use of relations of type (4) becomes problematic, since the obtaining of the impulse transient function from non-linear equations or even from equations with varying coefficients entails insuperable mathematical difficulties. The discussion will be confined only to the first type of non-linearity attributable to the varying value $D(\tau)$, as the strongest one, which enables the protection of the results obtained from the effect of other linear factors in order to most clearly present the main points of the adaptive method. The equations for predicting the flow enthalpy are written assuming that the thermal physical properties of the flow are constant:

$$\Delta D \frac{di_0}{dz} + D \frac{\partial \Delta i}{\partial z} + f \rho_0 \frac{\partial \Delta i}{\partial \tau} = \Delta \alpha h(\theta_0 - t_0) + \alpha h(\Delta\theta - \Delta t)$$

$$\Delta q - g_M c_M \frac{\partial \Delta \theta}{\partial \tau} = \Delta \alpha h(\theta_0 - t_0) + \alpha h(\Delta\theta - \Delta t). \quad (1a)$$

Here $\rho_0 =$ const. and α is taken to be a function of D alone. At $\Delta D =$ const. equation (1a) describes the system with constant parameters. The specific feature of the perturbation from $\Delta D(\tau)$ is the fact that, like the inlet effect, it induces a change in the variables and parameters of the system, e.g. in $D(\tau)$ and $\alpha(\tau)$. Therefore, its impulse function depends only on the form of the inlet signal $\Delta D(\tau)$, i.e. it is non-linear. For a single perturbation the convolution integral takes on the following form:

$$\Delta i(z,\tau) = \int_0^\tau E_{iD}[z,\tau-t,D_0+\Delta D(t)]\Delta D(t)\,\mathrm{d}t. \quad (7)$$

Here the stationary components of the vector $\boldsymbol{\sigma}$ are omitted. The impulse characteristics belong to the class of finite functions, for which the concept of the effective duration T_{eff} is introduced (i.e. the concept of such a quantity that $|E(\tau_1)| < \varepsilon$ for any $\tau_1 > T_{\text{eff}}$, where ε is a negligible number). Thus, without a great loss of accuracy, the lower integration limit in expression (7) can be replaced by $\tau-\tau_{\text{eff}}$.

Assuming that the 'drift' of the non-linear impulse transient function parameters is rather slow on the interval $[\tau-\tau_{\text{eff}}, \tau]$, the method of the 'frozen' impulse transient function can be used. The essence of the method is that for each value of time τ the non-linear function E_{iD} is approximately substituted by a linear function determined at a certain fixed value D^*. In the general case D^* should take on the mean value of $D(\tau-T_{\text{eff}})$ and $D(\tau)$. By selecting one of these limiting values, e.g. $D^* = D(\tau)$, it is possible to change over from equation (7) to the formula

$$\Delta i(z,\tau) = \int_{\tau-T_{\text{eff}}}^{\tau} E_{iD}[z,\tau-t,D(\tau)]\Delta D(t)\,\mathrm{d}t. \quad (8)$$

Here E_{iD} is defined by expression (3), in which at each new value of τ there occurs the rearrangement of the coefficients depending on $D(\tau)$. In much the same manner integral relations are obtained for other perturbation-transmitting channels. A successive averaging of the remaining coefficients of the differential equations in the course of dynamic calculations is carried out with allowance for the transient character of the thermal physical parameters.

The integration in equation (8), just as in equation (4), can be performed analytically using, for instance, the spline approximation of the function $\Delta D(\tau)$, which is generally of arbitrary form.

To qualitatively estimate the theoretical approximation, the physical experiment was carried out in a channel with a heat releasing envelope (Fig. 3). The channel of tubular geometry has an inner diameter of 11 mm and a wall thickness of 3.5 mm. The embedded sheathed thermocouples made of 0.27 mm diameter thermocouple wire were placed in the initial section and along the channel at distances of 6.12, 12.74 and 18.86 m from the entrance. The flow rate was measured with the aid of a quick-response sensor of tachometer type. By moving the regulating device the flow redistribution between the experimental section and bypass was conducted without altering the total pressure in the section.

Three forms of disturbances were considered (Fig. 4). The experiments were performed with water at the following regime parameters: $\rho_w = 680$–2400 kg m^{-2} s^{-1}, $p = 8.6$–9.2 MPa and $q = 4$–6 kW m^{-1}. The inertial properties of the dynamic system are given by the plots of the impulse characteristics presented in Fig. 5. Comparison of the theoretical results obtained from formula (8) with the experimental data revealed their good agreement for the monotonous decrease or increase (Fig. 6). It can be seen from the plots that the flow rate variations for the time interval of the order of the effective duration of the impulse transient function were fairly large.

A good accuracy can also be presented at the high-amplitude flow rate oscillations if the period of oscillations exceeds T_{eff}. The conditions of the method do not allow its use for the process induced by short-duration high-amplitude disturbances. One of the plots in Fig. 7 is obtained at a short two-fold increase in the flow rate, where the duration of the impulse upper plateau was shorter than T_{eff}. Since for the T_{eff} time the flow rate undergoes two sign-alternating high-amplitude perturbations, the discrepancy between theory and experiment is observed in the second part of the transient process. This discrepancy diminishes on a decrease in the perturbational amplitude or an increase in the perturbing impulse duration.

The method of step transient function (STF)

Consider another form of the convolution integral:

$$\Delta i(z,\tau) = \int_0^\tau h_{ix}(z,\tau-t,\boldsymbol{\sigma})\,\mathrm{d}\Delta x(t). \quad (9)$$

Unlike linear relation (4), here the vector $\boldsymbol{\sigma}$ is non-stationary and depends on the perturbing functions. First it will be assumed that the flow rate variation is the sole perturbation affecting the system. Allowing for this fact, equation (9) will be written in discrete form as:

$$\Delta i(z,\tau) = \sum_{m=1}^{K} h_{iD,m}(z,\tau-\tau_m,\boldsymbol{\sigma})\Delta D_m$$

$$\tau_K \leqslant \tau < \tau_{K+1}. \quad (10)$$

The variation $D(\tau)$ is approximated by the function of piecewise constant form (Fig. 8), in which the time step $\Delta\tau = \tau_{m+1}-\tau_m$ will be assumed to be uniform for convenience. This condition simplifies the computational scheme but it is not obligatory. During approximation a time delay by τ_1 can appear; in the absence of this delay τ_1 is assumed to be equal to zero. The introduction of discrete variations in the system parameters makes it possible to construct an analytical solution to the problem.

By solving equations (1a) on the interval $[\tau_1, \tau_2]$ under zero initial conditions, the step transient function $h_{iD,1} = h_{iD,1}(z,\ \tau-\tau_1,\ D_0,\ D_1)$ is found. The expression obtained has the form [3]:

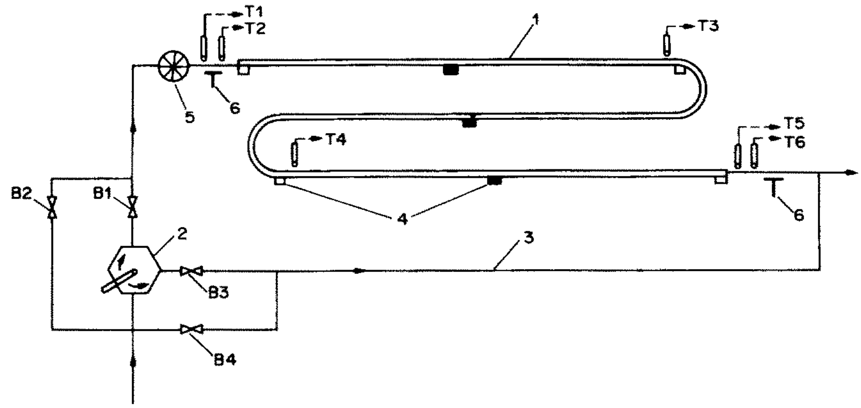

FIG. 3. Scheme of the experimental channel. 1, Heated section; 2, distributor; 3, bypass; 4, current leads; 5, tachometric flow rate pick-up; 6, pressure pick-up. B1–B4, Regulating valves; T1–T6, embedded thermocouples.

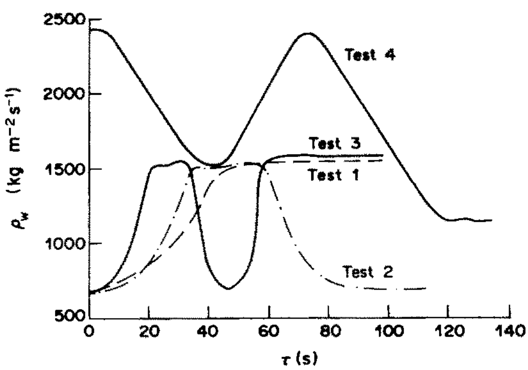

FIG. 4. Examples of flow rate (mass flow rate) disturbances.

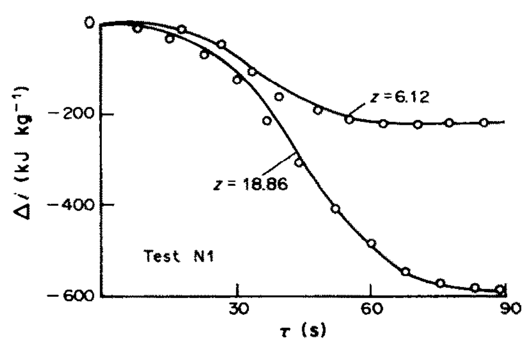

FIG. 6. Comparison of the prediction (the FITF method) with experiment in the case of a smooth increase in the flow rate. (——) Theory; (○), experiment.

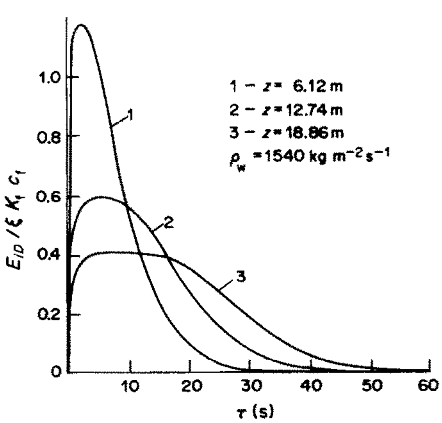

FIG. 5. The impulse transient function E_{iD} for a single-phase heat transfer agent (water) in different sections of the channel.

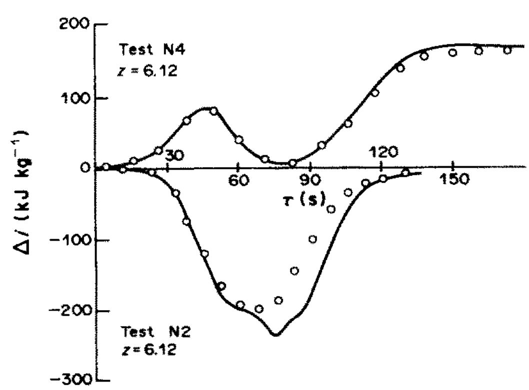

FIG. 7. Comparison of the prediction (the FITF method) with experiment in the case of non-monotonous variations in the flow rate. (——) Theory; (○), experiment.

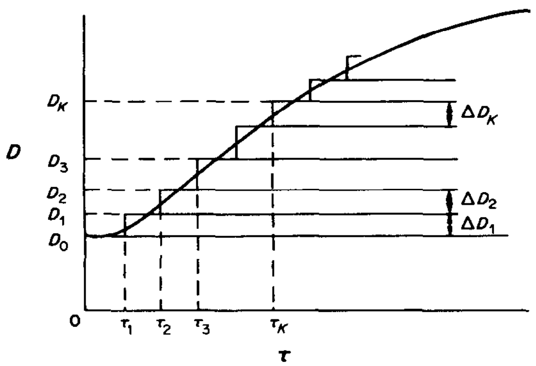

FIG. 8. Approximation of $D(\tau)$ by the step function.

$$h_{iD}(z,\tau) = \frac{c}{c-1}K_B c_B[\phi_1+(1-n)\phi_0]. \quad (11)$$

Here

$$\phi_0 = 1-e^{-s_0\tau}-V_1+V_{1,c}$$

$$\phi_1 = \frac{\tau}{T_M}-\frac{1}{1-c}\phi_0-V_2$$

and the arguments of the V functions are $\xi = z\alpha_1 h/D_1 c_B$, $\eta = (\tau-\xi T_B)/T_M$.

It is seen that the values of ξ and η are determined from the perturbed value of the flow rate D_1. The initial value of the flow rate is then presented in terms of K_s.

When $\Delta\tau > T_{eff}$, the transient process due to the step change in the flow rate by ΔD_1 is nearly accomplished by the time τ_2. The calculations on the interval $[\tau_2, \tau_3]$ should be continued from the achieved intermediate stationary state, which corresponds to the combination of the parameters at $\tau = \tau_2$; this gives the next step transient function $h_{iD,2} = h_{iD,2}(z, \tau-\tau_2, D_1, D_2)$. If $\Delta\tau < T_{eff}$, account should be taken of the non-stationarity induced by the preceding step ΔD_1. Mathematically, this leads to problem formulation with non-stationary initial conditions. However, a direct way of solving this problem turns out to be inefficient because of the extremely complicated resulting expressions to be operated at each $\Delta\tau$.

A simpler but basically approximate approach will be considered. Relations of type (10) are constructed by the principle of superposition of individual components of the total responses, thus essentially simplifying the computational procedure. For predicting the enthalpy variation Δi over the interval $[\tau_2, \tau_3]$, the following approximate expression can be written:

$$\Delta i(z,\tau) = h_{iD,1}(z,\tau-\tau_1,D_0,D_1)\Delta D_1 + h_{iD,2}(z,\tau-\tau_2,D_1,D_2)\Delta D_2,$$
$$\tau_2 \leqslant \tau < \tau_3. \quad (12)$$

It should be noted that this relation is non-linear because of the dependence of the step transient function parameters on the disturbance. Similarly, superposition can be constructed at any number of disturbances. Assuming, for instance, that the dynamic process is conditioned by the simultaneous change in the flow rate and heat input, the following expression can be obtained:

$$\Delta i(z,\tau) = h_{iD,1}(z,\tau-\tau_1,D_0,D_1,q_0)\Delta D_1 + h_{iq,1}(z,\tau-\tau_1,D_0)\Delta q_1 + h_{iD,2}(z,\tau-\tau_2,D_1,D_2,q_1)\Delta D_2 + h_{iq,2}(z,\tau-\tau_2,D_1)\Delta q_2,$$
$$\tau_2 \leqslant \tau < \tau_3. \quad (13)$$

As seen from this expression, in the course of rearranging the step transient function parameters, allowance is made for the cross effect of two different disturbances on the resulting variation Δi.

It follows from a more rigorous analysis conducted via replacing equations (1a) by the model with lumped parameters (an exact analytical solution of the non-linear dynamic problem is obtained for it [10]) that the first terms in equations (12) and (13) should contain an exponential cofactor which would correct the continuation of solutions $h_{iD,1}$ and $h_{iq,1}$ over $[\tau_2, \tau_3]$ with allowance for the changed conditions of the process. This factor does not influence the magnitude of total dynamic increments. The question of refining the approximate expressions will not be discussed here. Emphasis will be on the results which can be obtained from relations of types (12) and (13) presented in the full form

$$\Delta i(z,\tau) = \sum_{m=1}^{K} h_{iD,m}(z,\tau-\tau_m,D_{m-1},D_m,q_{m-1})\Delta D_m + \sum_{m=1}^{K} h_{iq,m}(z,\tau-\tau_m,D_{m-1})\Delta q_m,$$
$$\tau_K \leqslant \tau < \tau_{K+1}. \quad (14)$$

It is not difficult to track in a similar fashion the enthalpy perturbations of the incoming flow and the pressure in the channel. The corresponding solutions [3] have the following form

$$h_{ii} = V_1$$

$$h_{iq} = \frac{c}{c-1}K_q c_B \phi_1$$

$$h_{ip} = \frac{c}{c-1}K_p c_B \phi_0. \quad (15)$$

The computational results obtained using formula (14) are presented in Figs. 9 (only the flow rate is perturbed) and 10 (the simultaneous perturbation of $D(\tau)$ and $q(\tau)$). Here the results of the experimental data processing are shown by dots. The capabilities of the step transient function method appear to be greater compared with the 'frozen' impulse transient function method. This is evident from comparison of Figs. 9(b) and 7(a) and also from the results obtained in the case of flow rate perturbations (Fig. 9(d)), fluctuating with

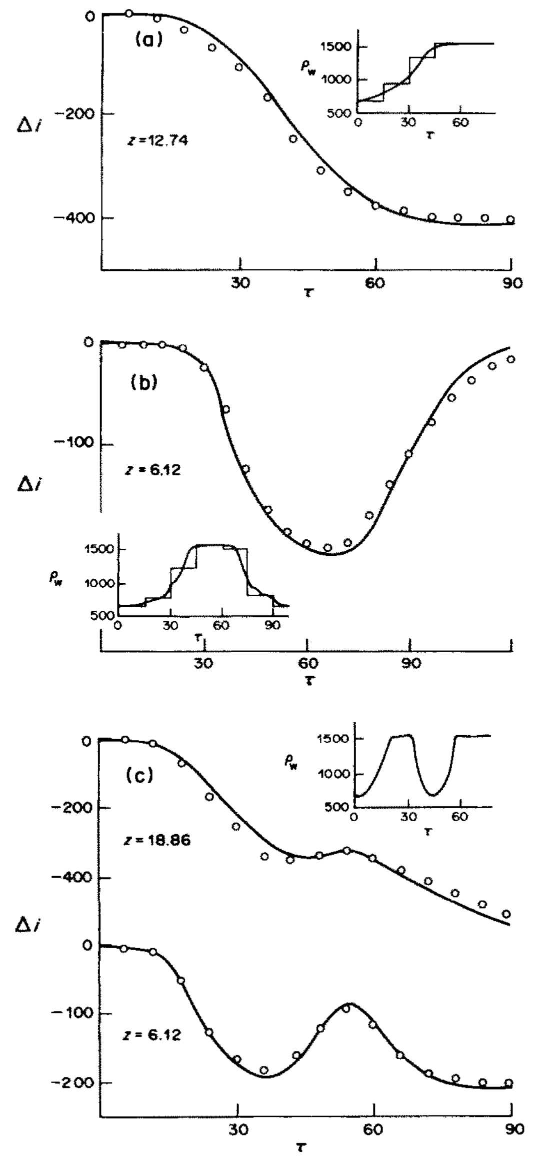

FIG. 9. Variations of the flow enthalpy in the course of flow rate disturbances. (——) Formula (14); (○), experiment.

a frequency sufficient for detecting the filtrating properties of the dynamic system. An attempt to employ the 'frozen' impulse transient function technique in the last case yielded an unsatisfactory result.

The STF method is not sensitive to the selection of a time step assigned in approximating the entrance effects; good results were obtained for fairly large steps. This allows a reduction in the number of terms on the right-hand side of equation (12), thus in the long run effecting a saving in computational operations. Simultaneously, a part of the dynamic terms pass with time to the stationary part of the solution. Thus, at q = const. the following limit is carried out

$$\lim_{\tau\to\infty} h_{iD,m}(z,\tau-\tau_m,D_{m-1},D_m,q)\Delta D_m = -\frac{qz}{D_{m-1}D_m}\Delta D_m.$$

It can be readily verified that in this case

$$\sum_{m=1}^{K} h_{iD,m}(z,\tau-\tau_m,D_{m-1},D_m,q)\Delta D_m = -\frac{qz}{D_0D_n}(D_n-D_0) + \sum_{m=n+1}^{K} h_{iD,m}(z,\tau-\tau_m,D_{m-1},D_m,q)\Delta D_m,$$

$$1\leqslant n<K,\quad \tau_K\leqslant\tau<\tau_{K+1}.$$

The constraint $\tau-\tau_n\geqslant T_{\mathrm{eff},n}$ is imposed on τ_n.

The same can be illustrated on the examples of the perturbations of $D(\tau)$, $q(\tau)$, etc. Since the analytical expressions for $\tau\to\infty$ (actually, for $\tau>T_{\mathrm{eff}}$) give accurate increments, the method enables the computation of processes of any durations without accumulating errors. The r.m.s. deviation from the experiment calculated by the formula

$$\delta(\tau)=\frac{1}{\Delta i_{\max}}\left[\frac{1}{n}\sum_{K=1}^{n}(\Delta i_{\mathrm{theor.}K}-\Delta i_{\mathrm{exp.}K})^2\right]^{1/2}$$

virtually (at least in the variants considered) does not

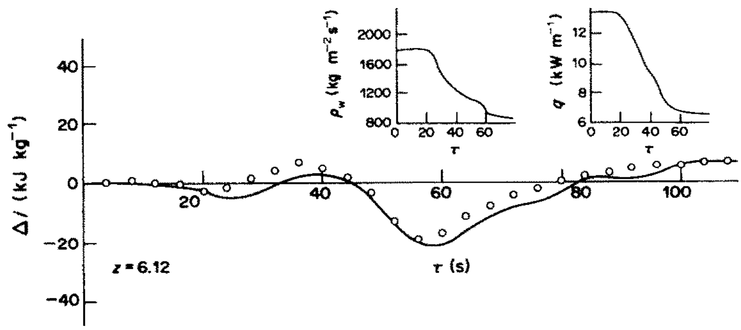

FIG. 10. Variations of the flow enthalpy with simultaneous disturbances of the flow rate and heat input. (——) Formula (14); (○), experiment.

exceed 5%, so the final stationary regime is achieved with absolute precision. It should be noted that the variation in the flow rate for the variant presented in Fig. 9 is able to induce the enthalpy variation by 480 kJ kg^{-1}.

CONCLUSION

The problems of analysing dynamic heat transfer processes in a channel with regard for non-linear effects are considered. The method chosen for the construction of approximate analytical solutions is based on the adaptive approach to the problem. Use is made of non-linear analogues of the convolution integral widely applied in setting up integral models of linear systems. Consideration is given to the possibilities for applying analytical expressions of the transient functions in the form of impulse and step characteristics which made it possible to obtain the simplest computational formulae for describing the dynamics of non-linear processes. The results of comparison with experiments are given in the course of the presentation. It is found that at slow but appreciable disturbances of arbitrary form the adaptive approach produces positive results; here the step transient function method is preferred.

The presentation is conducted on the example of finding the enthalpy dynamics for a single-phase flow; however, the flow rate dynamics can be predicted in the same manner which allows the application of the results obtained for solving the boundary-value problem of heat transfer dynamics in a channel by integral methods. In calculating the two-phase section, a more exact piecewise linear approximation of the perturbing functions is employed. For this case, the corresponding analytical solutions are also obtained, and are used in the non-linear integral model of a steam generating channel.

The problem of allowing for variations in the heat transfer agent thermal physical properties was not tackled here, since this can be done by the methods of mean-integral linearization in time.

REFERENCES

1. A. G. Shashkov, *Systematic Structural Analysis of Heat Transfer Processes and its Application.* Energoizdat, Moscow (1983).
2. J. Cermak, V. Peterka and J. Závorka, *Dynamika Regulovanych Soustav v Tepelne Energetice Chemii.* Česko-slovenska Academie Ved, Praha (1968).
3. E. P. Serov and B. P. Korol'kov, *Dynamics of Steam Generators.* Izd. Energiya, Moscow (1981).
4. B. P. Korol'kov and E. A. Tairov, The method of integral equations in the boundary-value problem of heat transfer dynamics, *Izv. AN SSSR, Energet. Transp.* No. 1, 82–88 (1984).
5. F. A. Vul'man and N. S. Khor'kov, *Computer Calculations of Thermal Power Equipment.* Izd. Energiya, Moscow (1975).
6. A. A. Armand, *Prediction of Transient Processes in Heat Exchangers. Increase of Steam Parameters and Power of Plants in Thermal Power Engineering.* Gosenergoizdat, Moscow (1961).
7. M. P. Simoyu, The realization of the nonlinear static mathematical model of a boiler unit by the successive integral linearization method, *Vopr. Prom. Kibernet.* **22,** 23–32 (1969).
8. B. P. Korol'kov, A. A. Pupin and E. A. Tairov, The selection of the method of averaging of thermophysical parameters when computing the dynamics of heat exchangers, *Izv. VUZov, Energetika* No. 7, 74–79 (1978).
9. B. P. Korol'kov, *Special Functions for Investigating the Dynamics of Unsteady Heat Transfer.* Izd. Nauka, Moscow (1976).
10. E. A. Tairov, Nonlinear simulation of heat transfer dynamics of the channel with single phase coolant, *Izv. AN SSSR, Energet. Transp.* No. 1, 150–156 (1989).

UNE METHODE NUMERIQUE–ANALYTIQUE POUR RESOUDRE LE PROBLEME NON LINEAIRE DU TRANSFERT DE CHALEUR DANS LES CANAUX

Résumé—Les façons de résoudre les problèmes non linéaires de la dynamique des mécanismes d'échange thermique sont discutées. Une approche numérique–analytique est suggérée à partir de l'analyse des possibilités des expériences physiques et numériques et des méthodes analytiques. Sont décrites deux méthodes numériques–analytiques (la méthode de "la fonction gelée de l'impulsion", FITF, et la méthode de "la fonction échelon transitoire", STF) en utilisant le concept d'adaptation pour construire une solution approchée pour le problème non linéaire par les relations fonctionnelles obtenues pour l'approximation linéaire. La méthode STF est justifiée expérimentalement et l'erreur des résultats théoriques n'excède pas 5%. La méthode FIFT donne aussi de bons résultats pour les processus monotones.

EINE NUMERISCH–ANALYTISCHE LÖSUNGSMETHODE FÜR NICHTLINEARE PROBLEME BEI DER INSTATIONÄREN WÄRMEÜBERTRAGUNG IN KANÄLEN

Zusammenfassung—Es werden die Lösungsmöglichkeiten nichtlinearer Probleme bei der instationären Wärmeübertragung untersucht. Ein numerisch–analytisches Näherungsverfahren wird vorgeschlagen, das durch die Untersuchung der Anwendungsmöglichkeiten physikalischer und numerischer Experimente sowie analytischer Methoden gewonnen wurde. Es werden zwei numerisch–analytische Verfahren ("frozen impulse-transient function" (FITF) und "step-transient function" (STF)) beschrieben, mit denen Näherungslösungen in Form von Funktionsausdrücken, die man durch lineare Approximation erhält, berechnet werden können. Die STF-Methode wird experimentell überprüft, der Fehler liegt dabei unterhalb von 5%. Mit der FITF-Methode sind gute Ergebnisse für monotone Vorgänge zu erzielen.

ЧИСЛЕННО–АНАЛИТИЧЕСКИЙ МЕТОД РЕШЕНИЯ НЕЛИНЕЙНОЙ ЗАДАЧИ ДИНАМИКИ ТЕПЛООБМЕНА В КАНАЛАХ

Аннотация—Обсуждаются пути решения нелинейных задач динамики обменных процессов. На основе анализа возможностей физического и численного эксперимента, а также аналитических методов предлагается численно–аналитический подход. Разработаны и описаны два численно–аналитических метода ("замороженной импульсной переходной функции' и "разгонной переходной функции"), позволяющие на основе функциональных соотношений, полученных для линейного приближения, построить приближенное решение нелинейной задачи, используя концепцию адаптации. Метод РПФ обоснован экспериментально, погрешность теоретических результатов не превосходит 5%. Для монотонных процессов хорошие результаты дает и метод "замороженной ИПФ".